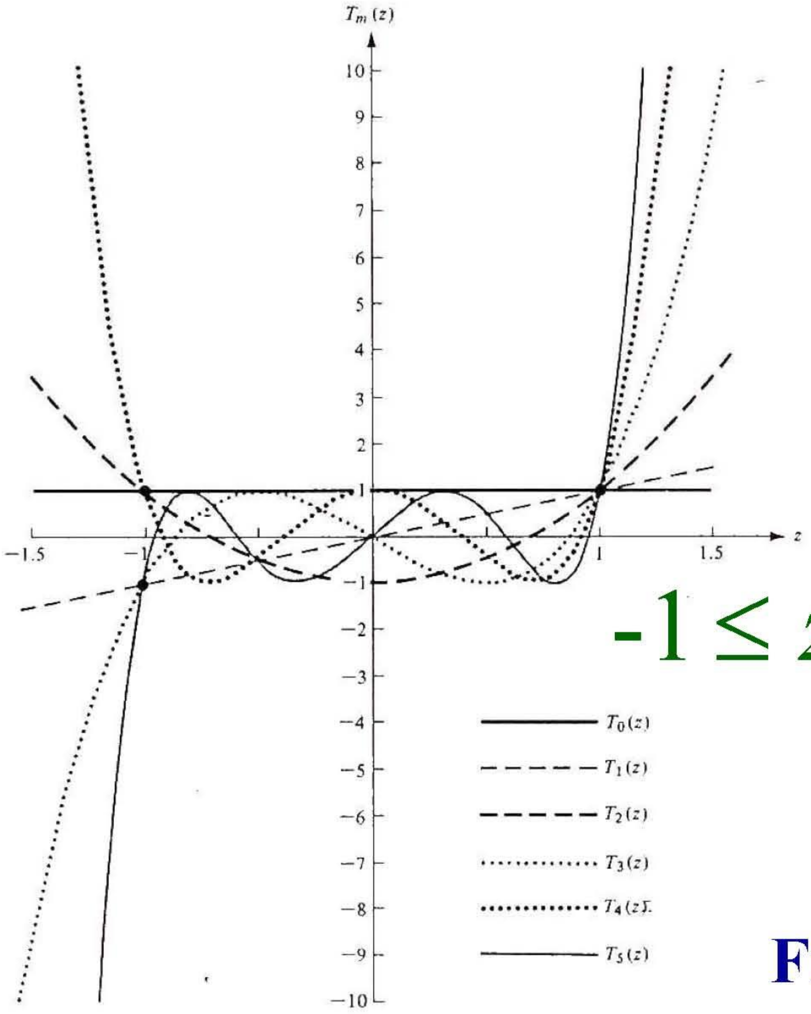
Dolph-Tschebyscheff Design

Equal Ripple; Chebyshev

Tschebyscheff Polynomials of Orders $n = 0 - 5$

$$T_n(z)$$



$$-1 \leq z \leq +1$$

Fig. 6.21

Tschebyscheff Polynomial of $n = 9$

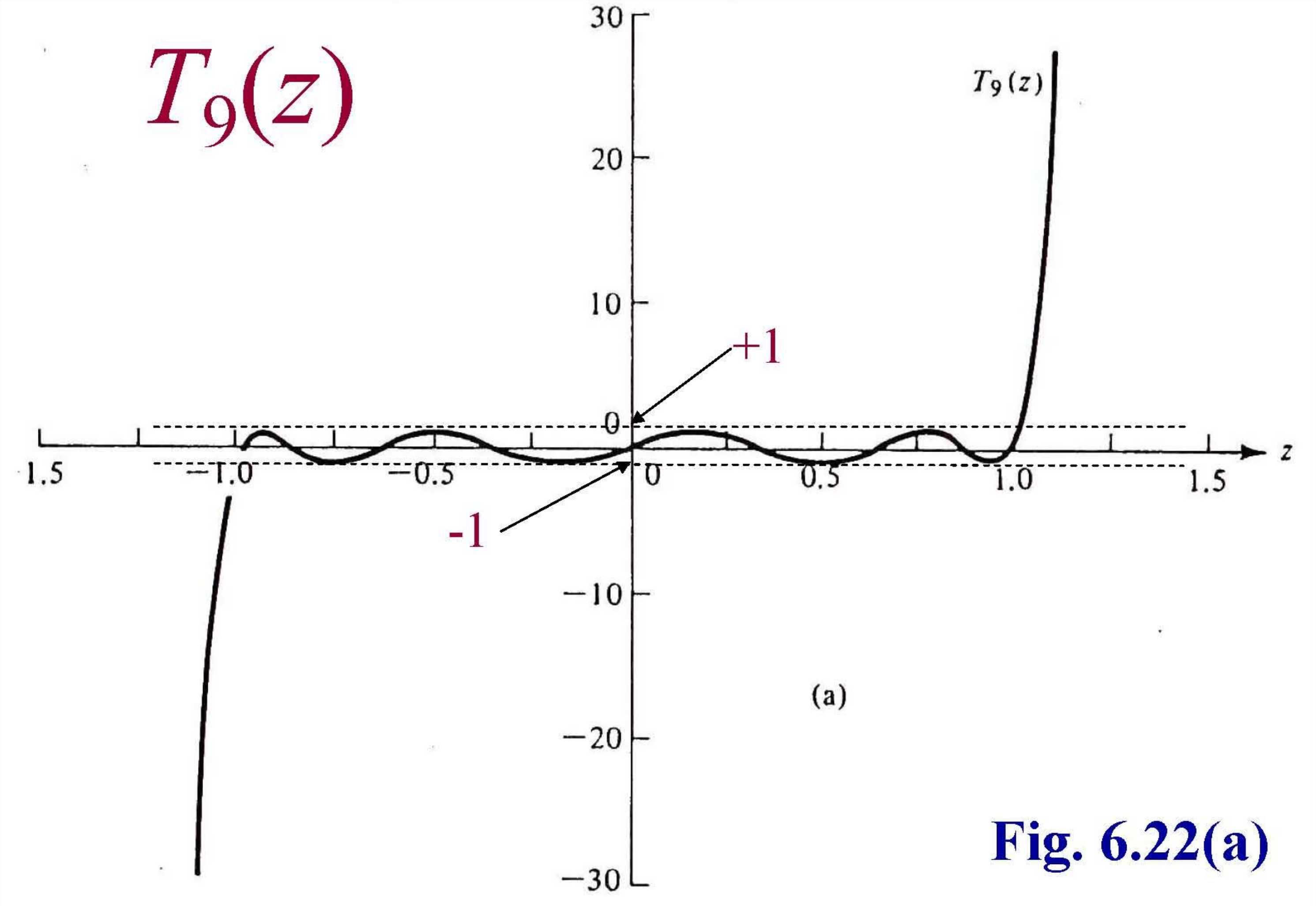


Fig. 6.22(a)

Magnitude of Tschebyscheff Polynomial of $n = 9$

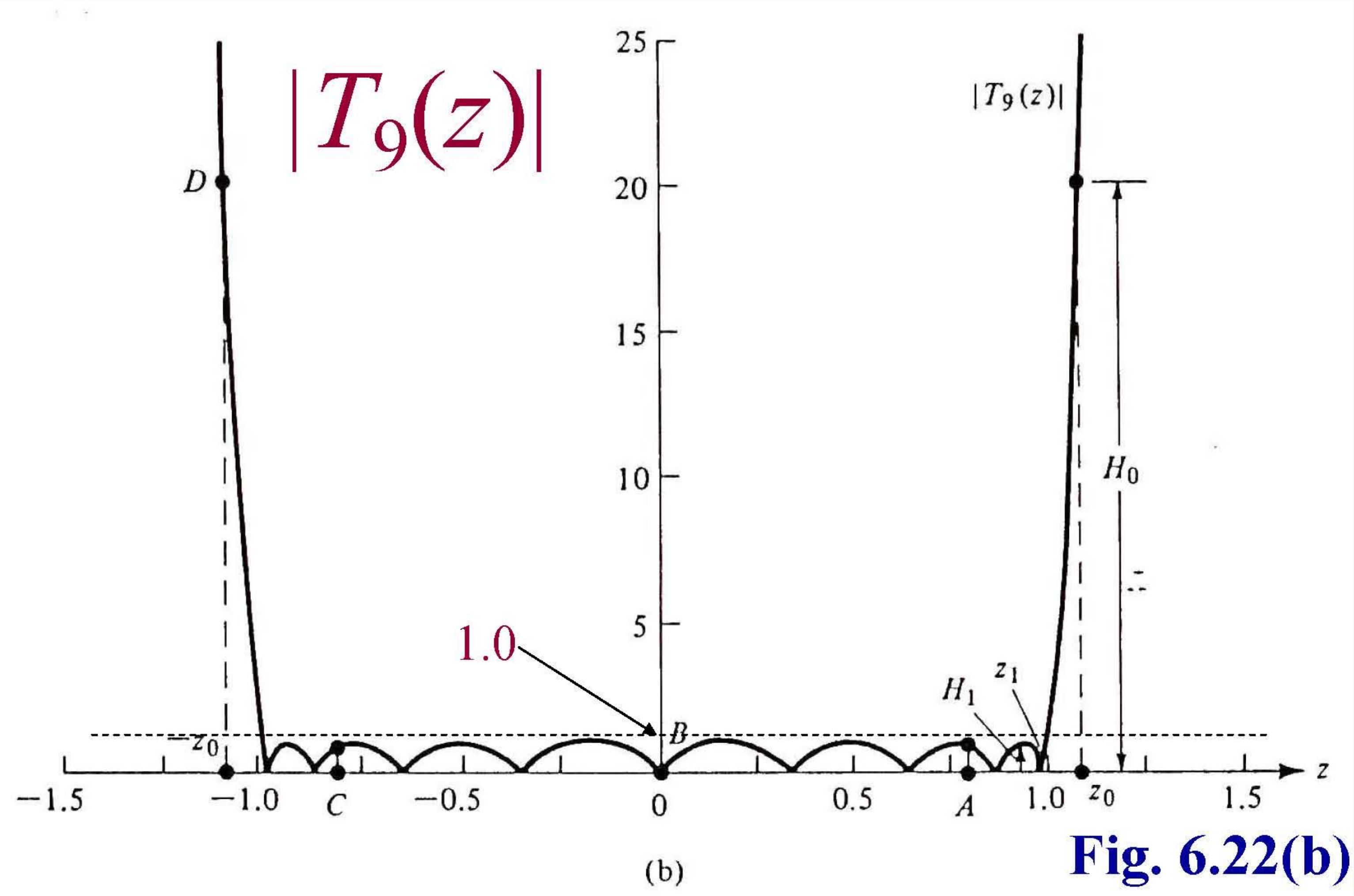


Fig. 6.22(b)

Broadside ($\beta = 0$) Dolph-Tschebyscheff

Design Procedure

1. Specify
   A. No. Of Elements ($2M$ or $2M+1$)
   B. Sidelobe Level (dB)
2. Find
   A. Amplitude Coefficients a_n
   B. Spacing d

Design (Synthesis) Procedure

1. Calculate $R_{\text{ovr}} \equiv$ Sidelobe Voltage Ratio

$$R_{ovr} = 10^{R_0(dB)/20}$$

2. Calculate p

$$p = \text{No. of Elements -1} = N\text{-1}$$

3. Calculate z_o

$$z_o = \cosh\left[\frac{1}{p}\cosh^{-1}(R_{ovr})\right]$$

or

$$z_o = \frac{1}{2}\left[\left(R_{ovr} + \sqrt{R_{ovr}^2 - 1}\right)^{1/p} + \left(R_{ovr} - \sqrt{R_{ovr}^2 - 1}\right)^{1/p}\right] \quad (6\text{-}73)$$

5. Select spacing d so that

$$d_{\max} \le \frac{\lambda}{\pi}\cos^{-1}\left(-\frac{1}{z_o}\right) \qquad \text{(6-76a)}$$

(if you want ***all*** the minor lobes to be of the same level)

$$
\begin{aligned}
m = 0 \quad & \cos(mu) = 1 \\
m = 1 \quad & \cos(mu) = \cos u \\
m = 2 \quad & \cos(mu) = \cos(2u) = 2\cos^2 u - 1 \\
m = 3 \quad & \cos(mu) = \cos(3u) = 4\cos^3 u - 3\cos u \\
m = 4 \quad & \cos(mu) = \cos(4u) = 8\cos^4 u - 8\cos^2 u + 1 \\
m = 5 \quad & \cos(mu) = \cos(5u) = 16\cos^5 u - 20\cos^3 u + 5\cos u \\
m = 6 \quad & \cos(mu) = \cos(6u) = 32\cos^6 u - 48\cos^4 u + 18\cos^2 u - 1 \\
m = 7 \quad & \cos(mu) = \cos(7u) = 64\cos^7 u - 112\cos^5 u + 56\cos^3 u - 7\cos u \\
m = 8 \quad & \cos(mu) = \cos(8u) = 128\cos^8 u - 256\cos^6 u + 160\cos^4 u - 32\cos^2 u + 1 \\
m = 9 \quad & \cos(mu) = \cos(9u) = 256\cos^9 u - 576\cos^7 u + 432\cos^5 u - 120\cos^3 u + 9\cos u
\end{aligned}
\tag{6-66}
$$

$$
z = \cos u \tag{6-68}
$$

(6-66) can be written as

$$
\begin{aligned}
m = 0 \quad & \cos(mu) = 1 = T_0(z) \\
m = 1 \quad & \cos(mu) = z = T_1(z) \\
m = 2 \quad & \cos(mu) = 2z^2 - 1 = T_2(z) \\
m = 3 \quad & \cos(mu) = 4z^3 - 3z = T_3(z) \\
m = 4 \quad & \cos(mu) = 8z^4 - 8z^2 + 1 = T_4(z) \\
m = 5 \quad & \cos(mu) = 16z^5 - 20z^3 + 5z = T_5(z) \\
m = 6 \quad & \cos(mu) = 32z^6 - 48z^4 + 18z^2 - 1 = T_6(z) \\
m = 7 \quad & \cos(mu) = 64z^7 - 112z^5 + 56z^3 - 7z = T_7(z) \\
m = 8 \quad & \cos(mu) = 128z^8 - 256z^6 + 160z^4 - 32z^2 + 1 = T_8(z) \\
m = 9 \quad & \cos(mu) = 256z^9 - 576z^7 + 432z^5 - 120z^3 + 9z = T_9(z)
\end{aligned}
\tag{6-69}
$$

$$
\boxed{T_m(z) = 2zT_{m-1}(z) - T_{m-2}(z)}
$$

Statement. Design a broadside Dolph-Tschebyscheff array of $2M$ or $2M + 1$ elements with spacing d between the elements. The side lobes are R_0 dB below the maximum of the major lobe. Find the excitation coefficients and form the array factor.

Procedure

a. Select the appropriate array factor as given by (6-61a) or (6-61b).

$$(\mathrm{AF})_{2M}(\text{even}) = \sum_{n=1}^{M} a_n \cos[(2n-1)u] \tag{6-61a}$$

$$(\mathrm{AF})_{2M+1}(\text{odd}) = \sum_{n=1}^{M+1} a_n \cos[2(n-1)u] \tag{6-61b}$$

where

$$u = \frac{\pi d}{\lambda}\cos\theta \tag{6-61c}$$

b. Expand the array factor. Replace each $\cos(mu)$ function ($m = 0, 1, 2, 3, \ldots$) by its appropriate series expansion found in (6-66).

c. Determine the point $z = z_0$ such that $T_m(z_0) = R_0$ (voltage ratio). *The order m of the Tschebyscheff polynomial is always one less than the total number of elements.*

d. Substitute

$$\cos(u) = \frac{z}{z_0} \tag{6-72}$$

e. Equate the array factor from step b, after substitution of (6-72), to a $T_m(z)$ from (6-69).

f. Write the array factor of (6-61a) or (6-61b) using the coefficients found in step e.

Example 6.9

I. Specifications

1. N = 10 Elements (even)
2. Sidelobe Level: -26 dB

II. Design Binomial & Tschebyscheff Arrays

1. Find Excitation Coefficients
2. Spacing Between Elements

Example 6.9 (cont'd.)

Tschebyscheff Design

Given: $N = 10,\ R_o(\text{dB}) = +26$

Solution

1. $R_{ovr} = 10^{26/20} = 10^{1.3} = 20$

2. $p = 10 - 1 = 9$

3. $z_o = \cosh\left[\frac{1}{9}\cosh^{-1}(20)\right] = 1.085$

$$z_o = \frac{1}{2}\left[(20 + \sqrt{400-1})^{1/9} + (20 - \sqrt{400-1})^{1/9}\right] = 1.0851$$

Alternate Method to Find Amplitude Coefficients (N=10): Example 6.9

1. $(AF)_{10} = a_1\cos(u) + a_2\cos(3u) + a_3\cos(5u) + a_4\cos(7u) + a_5\cos(9u)$

$$= T_9(z) = 256z^9 - 576z^7 + 432z^5 - 120z^3 + 9z$$

2. Replace, using the trigonometric identities of (6-66):

$$\cos(3u) = 4\cos^3 u - 3\cos u$$

$$\cos(5u) = 16\cos^5 u - 20\cos^3 u + 5\cos u$$

$$\cos(7u) = 64\cos^7 u - 112\cos^5 u + 56\cos^3 u - 7\cos u$$

$$\cos(9u) = 256\cos^9 u - 576\cos^7 u + 432\cos^5 u - 120\cos^3 u + 9\cos u$$

3. Find z_o : $\Rightarrow$ $z_o = \cosh\left\{\frac{1}{9}\cosh^{-1}\left[R_o(VR)\right]\right\}$ or (6-73)

4. Substitute:

$$\cos(u) = \frac{z}{z_o}$$

5. Equate terms of equal z^n power to find $a_n s$.

4. Normalized $a_n^{'}s$

Tschebyscheff		Binomial
$a_1 = 1$	$a_1 = 2.798$	$a_1 = 126$
$a_2 = 0.890$	$a_2 = 2.496$	$a_2 = 84$
$a_3 = 0.706$	$a_3 = 1.974$	$a_3 = 36$
$a_4 = 0.485$	$a_4 = 1.357$	$a_4 = 9$
$a_5 = 0.357$	$a_5 = 1$	$a_5 = 1$

$$d_{\max} \leq \left(\frac{\lambda}{\pi}\right)\cos^{-1}\left(-\frac{1}{z_o}\right)$$

$$z_o = \frac{1}{2}\left[\left(R_o + \sqrt{R_o^2 - 1}\right)^{1/p} + \left(R_o - \sqrt{R_o^2 - 1}\right)^{1/p}\right]$$

$R_o = +26$ dB $= 20(VR)$, $p = 10\text{-}1 = 9$

$\Rightarrow z_o = 1.0851$

$d_{\max} = 0.8731\lambda$

Array Factor Power Pattern of a 10-Element Broadside Dolph-Tschebyscheff Array

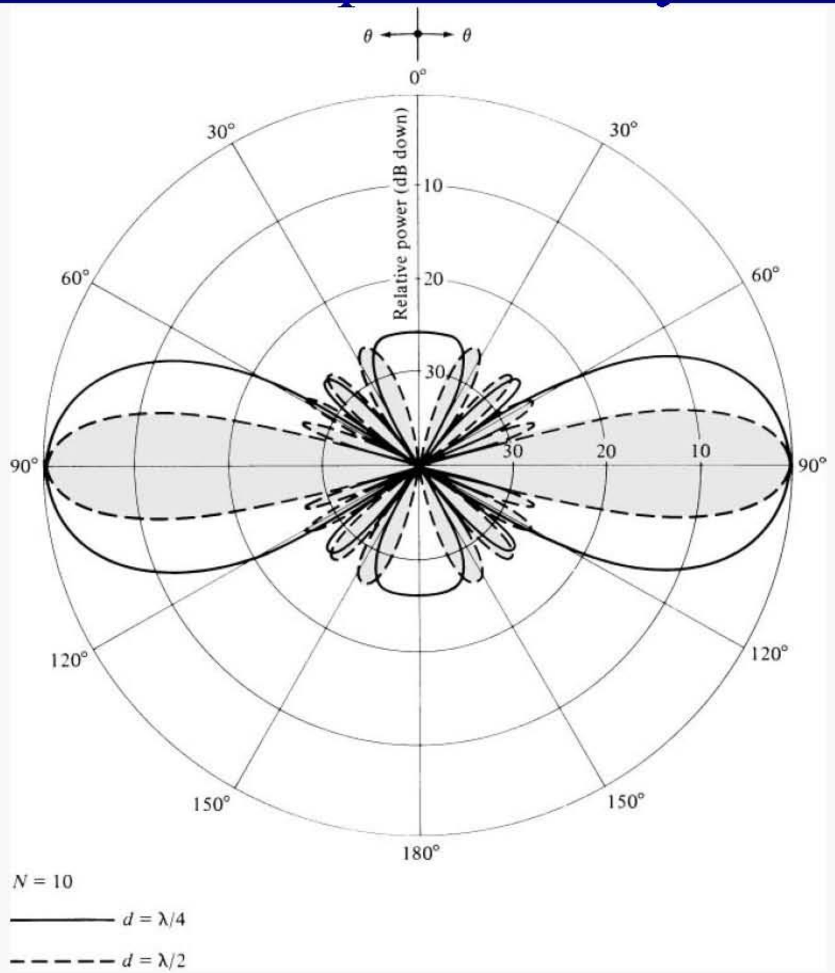


Fig. 6.23

Dolph-Tschebyscheff Arrays

A. Beam Broadening Factor

$$f = 1 + 0.636\left\{\frac{2}{R_o}\cosh\left[\sqrt{(\cosh^{-1} R_o)^2 - \pi^2}\right]\right\}^2 \tag{6-78}$$

B. Directivity

$$D_o = \frac{2R_o^2}{1 + (R_o^2 - 1) f \dfrac{\lambda}{(L+d)}} \tag{6-79}$$

Beam Broadening Factor

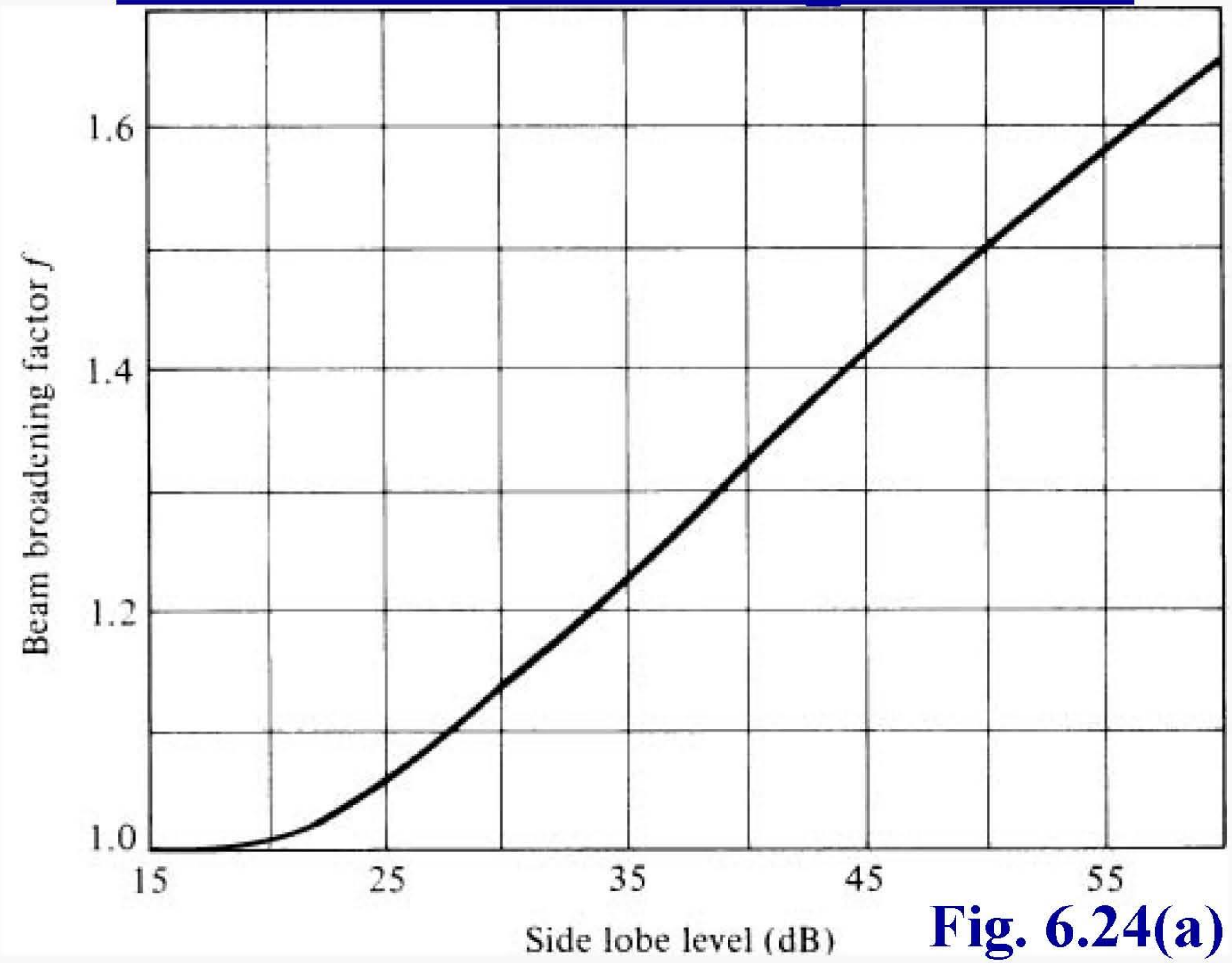


Fig. 6.24(a)

Directivity
of Tschebyscheff Arrays

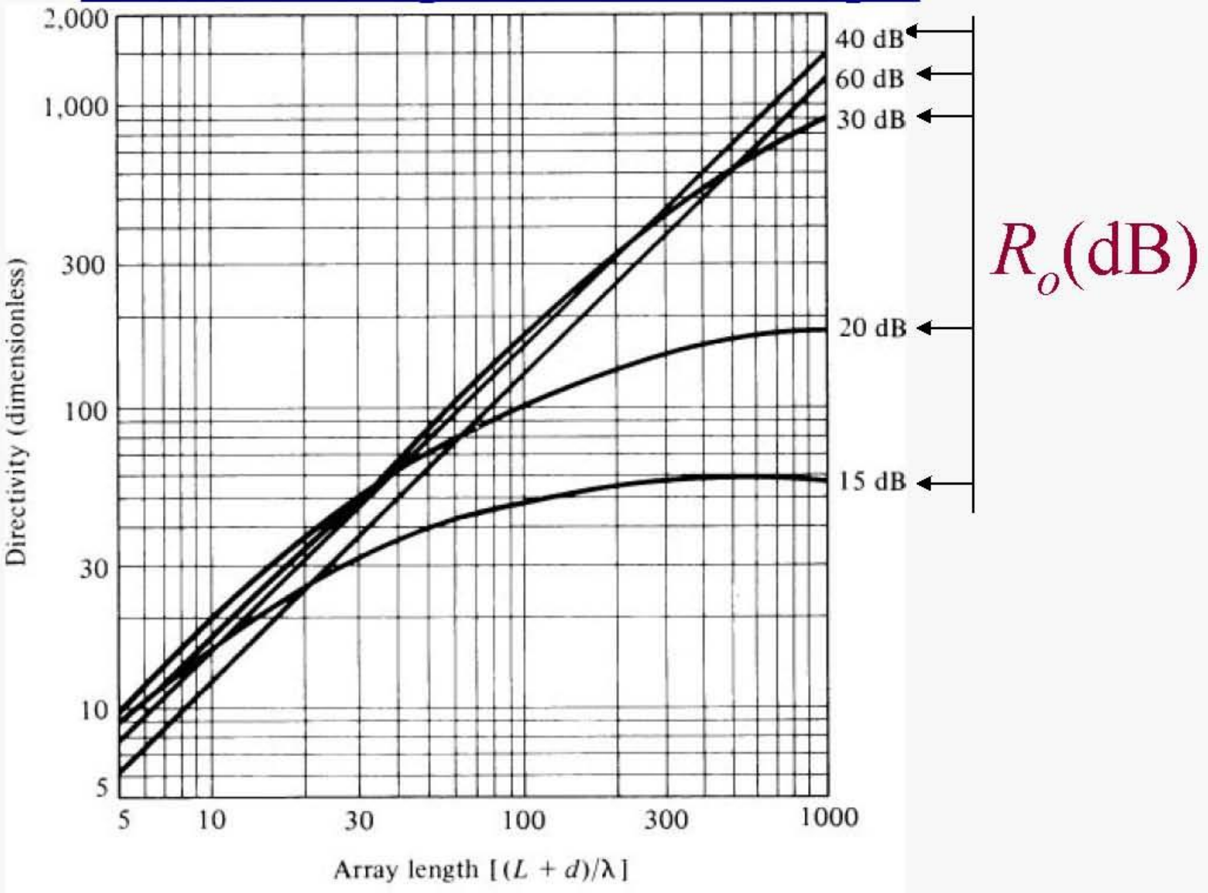


Fig. 6.24(b)

Example 6.10 :

Given: $N = 10,\ R_0(dB) = 26,\ d = \lambda/2$

Find: Θ_h and D_0

Solution:

From Example 6.7 $\Rightarrow R_o(VR) = 20$

$$f = 1 + 0.636\left\{\frac{2}{R_0(VR)}\cosh\left[\sqrt{\left(\cosh^{-1} R_0(VR)\right)^2 - \pi^2}\right]\right\}^2$$

$$= 1 + 0.636\left\{\frac{2}{20}\cosh\left[\sqrt{\left(\cosh^{-1} 20\right)^2 - \pi^2}\right]\right\}^2 = 1.079$$

$$\Theta_h(uniform) = \cos^{-1}\left[\cos\theta_0 - 0.443\frac{\lambda}{L+d}\right] - \cos^{-1}\left[\cos\theta_0 + 0.443\frac{\lambda}{L+d}\right]_{\substack{d=\lambda/2 \\ L=4.5\lambda}}$$

$$\Theta_h(uniform) = 10.17^\circ$$

$$\Theta_h(Tschebyscheff) = 10.17^\circ(1.079) = 10.97^\circ$$

Tschebysheff:

$$D_0 = \frac{2R_0^2(VR)}{1+\left[R_0^2(VR)-1\right]f\dfrac{\lambda}{L+d}}\Bigg|_{\substack{f=1.079 \\ R_0(VR)=20 \\ d=\lambda/2,\ L=4.5\lambda}}$$

$$D_0 = \frac{2(20)}{1+\left(20^2-1\right)1.079\dfrac{\lambda}{4.5\lambda+0.5\lambda}} = 9.18 = 9.63 \text{ dB}$$

Uniform:

$$D_0 = 2N\left(\frac{d}{\lambda}\right) = 2(10)(0.5) = 10 = 10 \text{ dB}$$

<u>N=10, d=λ/2: Uniform Array</u>

$$\Theta_h = 10.17^\circ$$

Sidelobe level @ -13.5 dB

<u>N=10, d=λ/2: Tschebyscheff Array</u>

$$f = 1.079 \Rightarrow \Theta_h = 1.079(10.17) = 10.97^\circ$$

Sidelobe level = -26 dB